The Market for Ideas and Economic Growth

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Abstract

This article studies the effects of intellectual property rights (IPR) and antitrust policies on aggregate innovation in an endogenous growth model. I show that public policies usually face a trade-off between innovation from incumbent firms and innovation from independent inventors. If inventors only innovate to enter product markets, the trade-off can lead to an inverted-U relationship between the economic growth rate and inventors' ability to benefit from their innovations. However, a strong IPR policy that protects inventors from incumbent firms on the market for ideas always increases aggregate innovation. First, an IPR policy has a stronger effect on incentives of inventors to innovate than it does on incentives of incumbent firms to do so. Second, the IPR policy increases the economic growth rate by encouraging inventors to innovate in order to sell innovations on the market for ideas instead of innovating in order to enter product markets.

1 Introduction

The expansion of the market for ideas in recent decades suggests that the interactions of incumbent firms and independent inventors often involves more than competition on product markets. In addition to pursuing innovations that enable inventors to replace current market leaders, inventors often pursue innovations that add complementary value to a market leader's technology and sell those innovations on the market for ideas. Here, inventors can be startups, research teams, or entrepreneurs. For example, a large part of technological progress in the pharmaceutical and information technology (IT) industries comes from innovations sold on the market for ideas through transactions such as technology licensing, contract research and development ($R\&D$), and mergers and acquisitions (M&A).

In this paper, I analyze the effects of policies on intellectual property rights (IPR) and antitrust actions on innovation when inventors can choose between two types of innovations. Inventors may pursue complementary innovations that add value to an incumbent's product that they subsequently try to sell to incumbents. Alternatively, inventors may pursue disruptive innovations that

would supersede the incumbent's product, replacing it in the product market. Specifically, I ask these questions: when do inventors pursue disruptive innovations? When do inventors focus on innovations that complement an existing value chain? How do different public policies impact which type of innovation an inventor opts to pursue? What consequences do different policies have on the innovation efforts of incumbents and inventors and, quite crucially, for the economic growth rate?

To facilitate faster economic growth, public policies need to consider two aspects of the innovation incentives of innovators. First, there may be a trade-off between incentives of incumbent firms and those of inventors that can hinge on the nature of innovations pursued by inventors. Accounting for the interplay of incumbents and inventors, I show how IPR policies and antitrust policies that protect inventors from incumbent firms may increase the innovation from inventors but decrease the innovation from incumbent firms. I identify how optimal policies to encourage economic growth hinge on the nature of inventors' innovations.

A second aspect that policy makers must consider is the type of innovation pursued by inventors. Complementary innovations are often more cost-efficient because inventors do not need to invest in the complementary assets and technologies that allow them to commercialize those innovations by themselves. In other words, inventors can avoid the costs of duplicating aspects of existing technologies and the costs of entry by pursuing complementary innovations. I show that if inventors' bargaining power on the market for ideas is sufficiently large, they switch from innovating to enter the product markets to innovating to sell on the market for ideas, and the resulting improvement in the efficiency of innovation activities leads to faster economic growth.

To study the effects of IPR and antitrust policies, I developed a continuous-time endogenous growth model that describes the interactions of incumbent firms with independent inventors and their investment decisions. There is a unit continuum of incumbent firms whose profits are proportional to the qualities of their products. An incumbent firm innovates to improve the quality of its product. There is also a mass of inventors that perform $R\&D$ whenever their net expected return is positive. In contrast to incumbents, an inventor chooses between the two types of innovations. An inventor can invest in market-oriented $R\&D$ to discover an innovation that improves an incumbent firm's product and then sell that innovation to the incumbent firm. Alternatively, an inventor can invest in entry-oriented $R\&D$ to seek to develop a new product of higher quality that would replace the incumbent firm's product. The instantaneous $R\&D$ investment of each innovator determines the Poisson arrival rate of its innovation. I assume that entry-oriented $R\&D$ is more costly than market-oriented $R\&D$ for the same innovation rate.

I solve for the unique linear balanced growth path (BGP) equilibrium. In

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this equilibrium, the equity value of an incumbent firm is proportional to the quality of its product. The equilibrium is characterized by the equity value function of incumbents, the extent of inventor entry, inventors' choice of innovation type, the aggregate innovation of incumbents, the aggregate innovation of inventors, and the expected payoffs to inventors from their innovations. I then show how the behavioral outcomes are determined by primitive parameters related to innovators' $R\&D$ technologies and different policies.

Public policies alter the allocation of the monopoly value of an inventor's innovation to its inventor and the related incumbent. Here, the monopoly value is the equity value of the incumbent using the innovation. An entrant's ability to capture the monopoly value of its innovation is called "appropriability" in literature. If a policy increases inventors' ability to benefit from their innovations, it decreases incumbent firms' expected payoffs. For example:

1. A policy that protects inventors' IPR increases inventors' bargaining power on the market for ideas and increases the expected payment from an incumbent to an inventor.

2. An IPR policy about blocking patents reduces the ability of incumbent firms to sue entrants for patent infringement, increases entrants' appropriability, and reduces the expected payment of an entrant to the related incumbent in the settlement.

3. An antitrust policy that reduces incumbents' ability to deter entry increases entrants' appropriability. If an entrant fails to survive, the related incumbent can purchase the entrant's innovation at a fire sale price. The policy reduces expected payoffs of incumbent firms when facing product market entrants.

I show that policies designed to promote innovation by inventors may reduce innovation by incumbents because of how they affect inventors' and incumbents' expected payoffs when they interact in the market for ideas or in product markets. If the policy increases the ability of inventors to benefit from their innovations, it increases the incentive inventors have to innovate. An incumbent firm's incentive to innovate depends positively on the incremental equity value of its innovation. If its equity value function decreases, so does the incremental equity value of an innovation. An expectation of lower expected payoffs from future interactions with inventors reduces the equity value function of incumbent firms. Thus, if a policy increases the equity value function of incumbents, it increases the incumbents' innovation incentive. Given the functional form of the equity value function, the incremental equity value of incumbents' innovation decreases as well and, hence, so does their incentive to innovate.

I then show that the relationship between aggregate innovation and the strength of policies that increases entrants' appropriability can be an inverted U-shape for a large set of parameter values. Inventors' expected payoffs from

entry-oriented innovations depend positively on the monopoly rents of their innovations and on the ability of entrants to appropriate those gains. If appropriability increases, the incumbent firms' expected payoffs when facing entrants decreases and so does the equity value. The negative effect on the equity value function feeds back to affect inventors' innovation incentives. As a result, the aggregate innovation of inventors increases at a slower rate when entrants' appropriability increases. When appropriability is small, strengthening policies will encourage inventors to innovate more significantly than it will discourage incumbent firms from innovating. As a result, aggregate innovation increases. When appropriability is high, the encouragement effect on inventors' innovation is small, causing the aggregate innovation to decrease.

The relationship is monotonically increasing when inventors' $R\&D$ technology is sufficiently efficient relative to that of the incumbents. Here, greater efficiency is associated with higher quality improvements of resulting innovations, or lower costs of $R\&D$. Intuitively, if entry-oriented $R\&D$ is efficient relative to incumbents' $R\&D$, the encouragement effect of a policy on the innovation of inventors outweighs the discouragement effect on the innovation of incumbents.

In contrast to the possibly U-shaped relationship between appropriability and aggregate innovation, I find that strong IPR protection that increases inventors' bargaining power on the market for ideas always increases aggregate innovation. First, when inventors and incumbents interact on the market for ideas, IPR protection always encourages aggregate innovation; the highest point on the inverted U-relationship is not attainable for an IPR policy, given inventors innovate to sell on the market for ideas. I prove that at the peak, the policy strength is at a level in which inventors capture more than the incremental equity value of their innovations. The inventors' expected payoff from their innovations is bounded by the incremental value of their innovations even when IPR protection is perfect. Because market-oriented innovations can only be used by incumbent firms, they cannot extract more than the incremental equity value of incumbent firms.

In addition, strong IPR protection for inventors can increase aggregate innovation by encouraging inventors to innovate to sell. Because entry-oriented $R\&D$ is more costly, inventors prefer market-oriented $R\&D$ even if the expected payoffs from resulting innovations are smaller. At the same time, incumbent firms' expected payoffs from interactions with inventors are larger when inventors perform market-oriented $R\&D$ than when inventors perform entry-oriented $R\&D$. Thus, if IPR protection rises to a point in which inventors switch from innovating to enter product markets to innovating to sell, the equity value of incumbents increases due to their expected payoffs when interacting with inventors. Incumbent firms and inventors react to the increased value function, causing aggregate innovation to rise.

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This article offers insights on the effects of public policies on economic growth. Segal and Whinston (2007) model a discrete-time economy that allows them to provide a thorough analysis of how the different strategic behaviors by incumbents affects the expected value of product-market entry. However, due to the complexity of studying aggregate innovation when innovators are heterogeneous, Segal and Whinston (2007) could not provide clean characterizations when incumbents and entrants both innovate. The negative impact of entry on incumbents' innovation incentives was first identified by Acemoglu and Cao (2015). Their continuous-time framework provides a simple framework to study aggregate innovation of heterogeneous innovators, but they do not evaluate the impacts of public policies on economic growth. I enrich the framework of Acemoglu and Cao (2015) by introducing the market for ideas and identify how the varying strengths of public policies affect the economic growth rate.

My model highlights the complex role of patent scope. As discussed in Merges and Nelson (1990), a patent's scope includes its ability to protect inventors from substitute technologies and its ability to block improved products. I find that an IPR protection policy that protects inventors from imitation by incumbents increases the growth rate of the economy. The importance of IPR protection on technological transactions has been discussed in Gans and Stern (2003), Arora and Gambardella (2010), and Spulber (2015); as well, Arora and Ceccagnoli (2006), among others, have empirically tested this premise in the literature. The role of the market for ideas in my model differs from that of Spulber (2013), where patent licensing allows the spread of new technology to multiple users. My insight is that innovating for the market for ideas yields non-destructive innovations that therefore maintain incumbents' innovation incentives.

My finding that there is an optimal probability that a patent can block follow-on innovations is consistent with the literature on cumulative innovation such as: Chang (1995); O'Donohue, Scotchmer, and Thisse (1998); and Hopenhayn, Llobet, and Mitchell (2006). The optimal blocking probability determines an optimal payoff of future inventors to earlier inventors in their models. In these papers, the focus is on the trade-off of encouraging an initial inventor to innovate for a new product and other inventors to innovate for improvements. In contrast, I focus on scenarios in which incumbent firms also innovate, as typically occurs in the real world. Thus, the question is not the optimal incentive of an inventor in the cumulative innovation process, but rather who should receive greater encouragement to innovate? The answer depends on the $R\&D$ technologies of different innovators.

2 Model

A. Preferences and Product Market Equilibrium

I develop a continuous-time, infinite-horizon model to study the dynamic interplay between innovation by incumbents and inventors. The economy is in continuous time and admits a representative household with the following logarithmic preference over a unique final good:

$$U = \int_0^\infty exp(-\rho t) \ln C,$$

where C is the household's consumption, and $\rho > 0$ is the discount factor that implies consumers' time preference. Competitive manufacturers produce the final good. It requires labor and a continuum $j \in [0, 1]$ of intermediate goods as inputs. Labor supply is constant at L. The production function of the final good is

$$Y = \frac{1}{1-\beta}(\int_0^1 q_j^\beta y_j^{1-\beta} dj)L^\beta,$$

where y_j is the quantity of intermediate good j, and q_j is its quality. Only the highest-quality version of each intermediate good is used in the production of the final good. Each intermediate good is supplied by a monopolist who can produce the good at its highest quality, and is produced with a linear technology with the final good. The production function of intermediate good $j \in [0, 1]$ is

$$y_j = (1 - \beta)q_j.$$

In the specification, the marginal cost is proportional to the quality of the product because it is more expensive to produce higher-quality products. The marginal cost is normalized to $1 - \beta$ to simplify the expression of equilibrium profits of intermediate good producers. Throughout the paper, I use the price of the final good as numeraire and normalize it to one.

Since the product market equilibrium is a standard result of the Schumpeterian growth model, I only present the essential steps here. Profit maximization of final good producers gives the demand function of intermediate good j:

$$y_j = q_j p_j^{-\frac{1}{\beta}} L,$$

where p_j is the price of the intermediate good j. I follow the standard assumption that intermediate good producers can charge the unconstrained monopoly price.[1] The maximization problem of incumbent firm $j \in [0, 1]$ gives its optimal

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[1] The same assumption is used in Acemoglu and Cao (2015) and Akcigit and Kerr (2018). It can be rationalized by a two-stage price bidding game for any step-size of quality improvements. In the first stage, the incumbent firm of intermediate good j and each potential entrant pay an infinitely small fee to enter the second stage. In the second stage, parties who have paid the fee in the first stage announce their prices for intermediate good j. In equilibrium, only the quality leader pays the fee in the first stage, and it can charge the unconstrained monopoly price.

price and quantity:

$$p_j = 1,$$
$$y_j = q_j L.$$

Thus the profit of monopolist j is

$$\pi_j = \pi q_j, \tag{1}$$

where $\pi \equiv \beta L$. Since an incumbent firm's profit is proportional to its goods' quality, even an incumbent firm has incentives to innovate.

The average quality of intermediate goods is given by

$$\bar{q}(t) \equiv \int_0^1 q_j(t) dj. \tag{2}$$

Substituting $x_j = q_j L$ into the final good production function yields

$$Y = \frac{1}{1-\beta} \bar{q} L. \tag{3}$$

The economic growth is driven by the average quality improvements of all intermediate goods.

B. Research and Development

There are two types of innovators, incumbents and a mass of independent inventors. Incumbents can improve the quality of their products through research and development ($R\&D$). Inventors can choose either to invest in technologies to improve incumbents' products or to develop higher quality products to replace incumbents' products. If an inventor has discovered an innovation add-on to an incumbent's product, it sells the innovation to the incumbent. If an inventor has innovated a higher quality substitute of an incumbent firm's product, the inventor becomes an entrant, and competes with the incumbent on the product market. All innovators invest in $R\&D$ in pursuit of uncovering an innovation that arrives according to a Poisson arrival rate that hinges on the $R\&D$ expenditure. In addition to the $R\&D$ cost, inventors also incur a fixed flow cost. If there is a positive mass of inventors performing $R\&D$, the free entry condition of inventors implies that their expected return from $R\&D$ is zero.

Innovation by Incumbents. If incumbent firm j spends

$$c_f(x_f) q_j(t) = \frac{1}{2} \delta_f x_f^2 q_j(t), \delta_f > 0 \tag{4}$$

units of the final good on $R\&D$, it has an instantaneous flow rate of innovation equal to x_f. The $R\&D$ cost is proportional to the quality of the intermediate

goods. This means that it is more expensive to improve a higher-quality product. An innovation by the incumbent improves the quality of its good from $q_j(t)$ to $(1+\lambda)q_j(t)$, where $\lambda > 0$. As I will show later, the equilibrium return to an incumbent's innovation is proportional to the quality of its good. The linearity in the incumbents' optimal problem cancels out, and therefore the optimal $R\&D$ innovation rate of an incumbent is constant in the stationary equilibrium.

Innovation by Inventors. There is a large mass of inventors who could perform $R\&D$. They choose between two types of projects: market-oriented (type m) $R\&D$ and entry-oriented (type e) $R\&D$, each of which leads to different commercialization methods. Market-oriented $R\&D$ only grants an innovation add-on to an existing product to an inventor; this restricts the inventor from entering the product market and competing with the incumbent. Therefore, the inventor must sell the innovation to the incumbent. With entry-oriented $R\&D$, an inventor develops all technologies required to produce an improved product. These technologies include process technologies and business methods to produce and deliver a new product. Although not all of the costs are $R\&D$ costs, in reality, they can be considered as the cost of economic experiments conducted by potential entrants needed to generate an entry-oriented innovation. To preserve tractability, I model all costs as $R\&D$ costs. The inventor enters the relevant product market upon success.

To achieve an innovation flow rate of x, inventors must incur $R\&D$ cost $c_i(x)\bar{q}(t)$ in terms of the final good. Here the subscript denotes the type of $R\&D$, either market or entry-oriented. The $R\&D$ cost is proportional to the average quality of intermediate goods. Thus, the $R\&D$ cost increases with the technology level. The potential innovation is applicable to one intermediate good, drawn from the uniform distribution over the interval $[0,1]$. This assumption allows the model to have a symmetric structure where the Poisson flow rate of innovations applicable to an intermediate good is the same for all intermediate goods. If an innovation of type i is applied to intermediate good j, it improves the quality from $q_j(t)$ to $(1+\mu)q_j(t)$, $\mu > 0$. The $R\&D$ cost function is quadratic,

$$c_i(x) = \frac{1}{2}\delta_i x^2, i \in \{m, e\}, \delta_i > 0 \tag{5}$$

where m and e denote market-oriented $R\&D$ and entry-oriented $R\&D$ respectively. Because entry-oriented $R\&D$ incurs the costs of duplicating existing technologies and setting up complementary assets, it requires a larger investment than market-oriented $R\&D$ for the same innovation rate. I make the following assumption:

Assumption 1: entry-oriented $R\&D$ is more costly than market-oriented $R\&D$:

$$\delta_e > \delta_m \tag{6}$$

In addition to $R\&D$ costs, inventors incur a positive fixed flow cost $\phi\bar{q}(t) > 0$.

In the baseline model, inventors incur all $R\&D$ costs before they know how their innovations will be applied. In practice, inventors may decide whether to make a greater investment to enter product markets after they have a complementary innovation. For tractability, I assume inventors make that decision ex-ante. In Section 5, I extend the model to the setup where inventors know the applications of their ideas before they make $R\&D$ project choices.

C.Post-Innovation Expected Payoffs

Inventors' expected payoffs from their innovations are decided by the commercialization environments and public policies. When inventors sell market-oriented innovations on the market for ideas, they negotiate a price with incumbent firms. With entry-oriented innovations, inventors compete with incumbent firms to be the monopoly supplier of an intermediate good. Public policies affect inventors' bargaining power on the market for ideas and entrants' appropriability on a product market. I now discuss the expected payoffs of inventors and incumbents from different types of innovations.

Market-Oriented Innovations. An inventor with a market-oriented innovation trades that innovation with an incumbent in return for a lump-sum payment. Consider a transaction between an inventor and incumbent firm j. Let $V(q_j)$ be the value function of incumbent firm j, whose product quality is q_j. Incumbent firm j's reservation value for a market-oriented innovation is $V(q_j)$. Therefore, the bargaining surplus is the incremental value of the innovation

$$V((1+\mu)q_j) - V(q_j).$$

Denote the bargaining power of the inventor by $s_m(\alpha_m) \in [0,1]$, where α_m is the strength of a public policy that protects the inventors from incumbent firm j in the market for ideas. $s_m(\alpha_m)$ increases with α_m. The inventor's payoff from the transaction is

$$s_m(\alpha_m)[V((1+\mu)q_j) - V(q_j)], \tag{7}$$

and incumbent firm j's payoff from the transaction is

$$(1 - s_m(\alpha_m))[V((1+\mu)q_j(t)) - V(q_j)] + V(q_j). \tag{8}$$

In the market for ideas, the relevant policy is usually an IPR policy that protects inventors' IP from imitation by incumbent firms. I will discuss how the policy affects $s_m(\alpha_m)$ in Section 4.

Because the application of a market-oriented innovation is randomly drawn from a uniform distribution, the expected return to market-oriented $R\&D$ is given by

$$E_{j \in [0,1]}\{s_m(\alpha_m)[V((1+\mu)q_j) - V(q_j)]\}.$$

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I will show that the equilibrium equity value $V(q_j)$ is proportional to q_j for any intermediate good j, and therefore the expected return is proportional to the average quality of intermediate goods $\bar{q}$. Because the $R\&D$ cost is also proportional to $\bar{q}$, the equilibrium market-oriented innovation rate is independent of $\bar{q}$.

Entry-Oriented Innovations. The monopoly value of an innovation is given by the equity value of the monopolist with the innovation. Public policies determine the allocation of the monopoly rent of an entry-oriented innovation to the related entrant and the incumbent. That is, there is no value loss in the post-innovation interaction between the incumbent and the entrant. Let $R_e(\alpha_e, q_j)$ denote the expected payoff of an entrant of intermediate good j, where α_e is the strength of a public policy that protects entrants from incumbent firms' strategic behaviors on product markets. The expected value of monopolist j when there is an entrant is

$$V((1+\mu)q_j) - R_e(\alpha_e, q_j). \tag{9}$$

The following scenario illustrates how a public policy affects the allocation of the monopoly value of entry-oriented innovations to incumbents and entrants:

Policy 1: An antitrust policy to restrict predatory activities. In practice, an inventor may succeed with an innovation, but a monopolist may engage in predatory activities that destroy the profit opportunity of the innovator. I model this predatory behavior as a probabilistic, entry-deterring process[2].

If an entrant develops a product, then with probability $1 - \alpha_e$, an incumbent can prevent that entrant from successfully entering the market and then expropriate all the resources in a fire sale. The expected payoff of an innovation applicable to intermediate good j to the entrant is

$$R_e(q_j(t)) = \alpha_e V((1+\mu)q_j(t)).$$

Monopolist j's expected payoff facing an entrant is

$$V((1+\mu)q_j) - R_e(q_j(t)) = (1-\chi)V((1+\mu)q_j(t)).$$

I will develop the model and solve the stationary equilibrium in the scenario described under Policy 1. But the framework can be applied to any scenario in which the expected payoff of an entrant is a weighted sum of both the monopoly and incremental values of its innovation. I will discuss another important public policy in Section 4.

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[2]One such predatory activity is limit pricing. In Segal and Whinston (2005), deep-pocket incumbent firms can use limit pricing to reduce entrants' profits. Because entrants may lack access to sufficient financial resources, low profits may trigger bankruptcy and an exit of entrants.

The expected return of entry-oriented $R\&D$ is given by:

$$E_{j \in [0,1]} R_e(\alpha_e, q_j).$$

Because $R_e(\alpha_e, q_j)$ is a constant fraction of the monopoly value of an entry-oriented innovation, the expected return of entry-oriented $R\&D$ is proportional to the average quality of intermediate goods. In the inventors' optimization problem, the linearity in return and cost cancels out, and therefore the entry-oriented innovation rate is independent of $\bar{q}$ for all inventors.

D. Equilibrium

Definition (Balanced growth path (BGP) equilibrium). A BGP equilibrium of the economy is represented by the following tuple for every intermediate good $j \in [0,1]$, and time t: incumbent firm j's innovation flow rate $x_f^*(j)$, inventors' $R\&D$ choice I^*, inventors' innovation flow rate x_i and inventors' aggregate innovation flow rate τ_i, where $i = I^*$, the average quality of intermediate goods $\bar{q}$, and the interest rate r, total output Y^*, the aggregate consumption of households C^* such that

1. $x_f^*(j)$ maximizes the equity value of monopolist j, $j \in [0,1]$;

2. I^* and x_i, where $i = I^*$, maximizes inventors' expected net return of performing $R\&D$;

3. if $\tau_i > 0$, where $i = I^*$, inventors' free-entry condition is satisfied;

4. the total output Y^* satisfies equation (3);

5. the representative household maximizes utility;

6. the sum of aggregate consumption of households C, the aggregate cost of intermediate goods, and the total $R\&D$ investment is the total out put Y^*;

7. r is constant, and Y^*, C^* and $\bar{q}$ grows at the same constant rate.

The maximization problem of the representative household yields the household's Euler equation:

$$\frac{\dot{C}}{C} = (r - \rho).$$

In equilibrium consumption, growth equals the gap between the interest rate, which is the required return on investment, and the rate of time preference. The aggregate growth rate of the economy is given by

$$g = \frac{\dot{\bar{q}}}{\bar{q}}.$$

Because the aggregate consumption and average quality of intermediate goods grow at the same constant rate, the interest rate is obtained as

$$r = g + \rho \tag{10}$$

Inventors' Optimization Problem. An inventor seeks to maximize its expected net return by choosing a $R\&D$ project, and the scale of investment $x_i(t)$, $i \in \{m, e\}$. I define $I \in \{m, e\}$ to be the inventor's choice of the type of $R\&D$ project. Given the average quality of intermediate goods $\bar{q}$, an inventor's optimization problem is:

$$\max_{I \in \{m,e\}} \{v(I)\}, \tag{11}$$

where

$$v(m) \equiv \max_{x_m} x_m E_{j \in [0,1]}[s_m(\alpha_m(s_m))(V((1+\mu)q_j) - V(q_j))] - c_m(x_m)\bar{q} \tag{12}$$

is its optimization problem conditional on $I = m$; and

$$v(e) \equiv \max_{x_e} x_e E_{j \in [0,1]}[R_e(\alpha_e, q_j)] - c_e(x_e)\bar{q} \tag{13}$$

is its optimization problem conditional on $I = e$.

An inventor's optimal $R\&D$ project is

$$I^* = \begin{cases} m, & \text{if } v(m) \geq v(e). \\ e, & \text{if } v(m) < v(e). \end{cases} \tag{14}$$

If $I^* = m$, an inventor chooses a Poisson arrival rate of market-oriented innovation to maximize its expected net return. The marginal benefit of increasing the innovation rate is the expected payoff of a market-oriented innovation. The optimal innovation rate satisfies the following first order condition:

$$E_{j \in [0,1]}[s_m(\alpha_m)(V((1+\mu)q_j) - V(q_j))] = c'_m(x_m)\bar{q}, \tag{15}$$

at which the marginal benefit of a higher innovation rate equals its marginal cost. Thus the optimal innovation rate of an inventor is

$$x_m^* = \frac{E_{j \in [0,1]}[s_m(\alpha_m)(V((1+\mu)q_j) - V(q_j))]}{\delta_m \bar{q}} \tag{16}$$

The same analysis implies that the optimal Poisson arrival rate when $I^* = e$ is

$$x_e^* = \frac{E_{j \in [0,1]}[R_e(\alpha_e, q_j)]}{\delta_e \bar{q}} \tag{17}$$

In equilibrium, if there is a positive mass of inventors performing $R\&D$, their expected instantaneous net return equals the fixed cost $\phi\bar{q}$. The inventors' free-entry condition is

$$\max_{I \in \{m,e\}} \{v(I)\} = \phi\bar{q} \tag{18}$$

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In equilibrium, the aggregate $R\&D$ activities of inventors yields the aggregate market-oriented innovation rate τ_m and the aggregate entry-oriented innovation rate τ_e. They are determinant numbers in the BGP equilibrium. Because the applicable intermediate good of an innovation is drawn from the uniform distribution, $\tau_i(i \in \{m, e\})$ is also the aggregate type i innovation rate for an intermediate good, where $i \in \{m, e\}$. If equation (15) is satisfied, inventors perform market-oriented $R\&D$, and there is no entry. In other words, $\tau_e = 0$. Otherwise $\tau_m = 0$.

Incumbent Firms' Optimization Problem Incumbent firm j whose technology level is q_j seeks to choose the Poisson arrival rate of its innovation conditional on τ_m, τ_e and the interest rate r. Its Hamilton-Jacobi-Bellman equation is

$$rV(q_j) - \dot{V}(q_j) = \pi q_j + \max_{x_f}\{x_f[V((1 + \lambda)q_j) - V(q_j)] - c_f(x_f)q_j\}$$

$$+\tau_m(1 - s_m(\alpha_m))[V((1 + \mu)q_j) - V(q_j)] + \tau_e[V((1 + \mu)q_j) - R_e(\alpha_e, q_j) - V(q_j)]$$
(19)

On the left-hand-side of equation (16), the term $\dot{V}(q_j)$ is the derivative of $V(q_j)$ with respect to t. If $V(q_j)$ is dependent on $\bar{q}$, the term is not zero. The maximization problem on the right-hand-side is the instantaneous optimization problem of incumbent firm j. By investing $c_f(x_f, q_j)$ units of the final good on $R\&D$, the instantaneous expected return is $x_f[V((1 + \lambda)q_j) - V(q_j)]$. The first order condition of the problem yields the equilibrium innovation rate of monopolist j:

$$x_f^* = \frac{V((1 + \lambda)q_j) - V(q_j)}{\delta_f q_j}$$
(20)

The second line of the RHS of equation (16) is the expected change of incumbent firm j's value due to inventors' innovations. If $\tau_m > 0$ and $\tau_e = 0$, then incumbent firm j purchases market-oriented innovations from inventors with the Poisson arrival rate τ_m. The expected payoff of incumbent firm j from a technological transaction is $(1 - s_m(\alpha_m))[V((1 + \mu)q_j) - V(q_j)] + V(q_j)$. When the transaction happens, the change in its value is $(1 - s_m(\alpha_m))[V((1 + \mu)q_j) - V(q_j)]$. If $\tau_e > 0$, and $\tau_m = 0$, entrants arrive with the rate τ_e. The expected payoff of monopolist j when there is an entrant is $V((1 + \mu)q_j) - R_e(\alpha_e, q_j)$, and therefore the change in its value compared with its original equity value is $V((1 + \mu)q_j) - R_e(\alpha_e, q_j) - V(q_j)$.

E. Solution for Equilibrium

Using different innovators' $R\&D$ decisions, I now derive the growth rate from the law of motion of qualities of intermediate goods. Given $I^* = i$, consider the

quality improvement of intermediate good j over a time interval of length dt,

$$q_j(t + dt) = \begin{cases} (1 + \lambda)q_j \text{ with probability } x_f^* dt \\ (1 + \mu_m)q_j \text{ with probability } \tau_m dt \\ (1 + \mu_e)q_j \text{ with probability } \tau_e dt \\ q_j \text{ with probability } 1 - x_f^* dt - \tau_m dt - \tau_e dt \end{cases} \tag{21}$$

There is no aggregate uncertainty, and the aggregate growth rate g is as described in the following lemma.

Lemma 1. In the BGP equilibrium, the aggregate growth rate is given by

$$g = x_f^* \lambda + \tau_i \mu \tag{22}$$

Lemma 1 implies that the economic growth rate is the sum of aggregate innovation by incumbents and inventors. The aggregate innovation by incumbents is $x_f^* \lambda$, which is the product of the innovation rate of incumbents and the step-size of quality improvement of their innovations. Similarly, the aggregate innovation by inventors is $\tau_i \mu$. Innovation rates x_f^*, τ_m and τ_e are endogenously determined in equilibrium.

I focus on the linear BGP, where the value function of a monopolist producer with quality q is proportional to q. Conjecture $V(q_j) = Aq_j$, where $A \geq 0$ is endogenous. I refer to A as the value of quality because it measures the average and marginal equity value of an intermediate good's quality. With the linear structure, the incremental value of an inventor's innovation applicable to intermediate good j is $\mu A q_j$.

Proposition 1: There is a unique linear BGP equilibrium. In this equilibrium, the value function of a monopolist takes the form

$$V(q) = A_i q,$$

where $i \in \{m, e\}$ denotes the nature of $R\&D$ that inventors pursue in equilibrium.

An entrant can appropriate fraction $s_e(\alpha_e)$ of the incremental value of its innovation. Using the linear form of incumbents' value function, I see that $s_e(\alpha_e)$ is an exogenous parameter determined by the step-size of the quality improvement of entry-innovations and the strength of the related policy. For Policy 1, with probability $1 - \alpha_e$, an inventor's entry to product market j is blocked, and the inventor has to sell its innovation in a fire sale. The expected payoff of the inventor is $\alpha_e V((1 + \mu)q_j)$. Thus s_e is given by

$$s_e(\alpha_e) = \frac{\alpha_e V((1 + \mu)q_j)}{V((1 + \mu)q_j) - V(q_j)} = \frac{\alpha_i(1 + \mu)}{\mu}. \tag{23}$$

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$s_e(\alpha_e)$ depends positively on the survival probability of entrants α_e. When $\alpha_e = 1$, inventors who have developed new products can always replace incumbent firms on product markets successfully, and $s_e(\alpha_e) = \frac{1+\mu}{\mu}$.

For inventors, the expected value of an $R\&D$ project is therefore

$$v(i) = \frac{1}{2\delta_i}(s_i(\alpha_i)\mu A)^2 \bar{q}, i \in \{m, e\} \tag{24}$$

Inventors choose market-oriented $R\&D$ if and only if $v_m \geq v_e$.

Lemma 4. In the linear BGP equilibrium, inventors choose market-oriented $R\&D$ if

$$\frac{s_m(\alpha_m)}{s_e(\alpha_e)} \geq \sqrt{\frac{\delta_m}{\delta_e}}. \tag{25}$$

If, instead,

$$\frac{s_m(\alpha_m)}{s_e(\alpha_e)} > \sqrt{\frac{\delta_m}{\delta_e}},$$

they choose entry-oriented $R\&D$, seeking to displace an incumbent firm.

When equation (25) holds, the expected value of an entry-oriented $R\&D$ project is smaller than that of a market-oriented $R\&D$ project. If inventors' bargaining power $s_m(\alpha_m)$ in the market for ideas is large enough, or entrants' appropriability in the product markets is small enough, inventors prefer market-oriented $R\&D$ to entry-oriented $R\&D$. Because by assumption 1, $\delta_m < \delta_e$, when equation (25) holds, the left-hand side of it may be less than 1. Inventors may choose market-oriented $R\&D$ even if the expected payoffs of market-oriented innovations are less than those of entry-oriented innovations to save $R\&D$ costs.

With equation (26), inventors' free-entry condition pins down the equilibrium value of quality A_i,

$$A_i = \frac{\sqrt{2\phi\delta_i}}{s_i(\alpha_i)\mu} \tag{26}$$

where $i = I^*$ is the type of $R\&D$ that inventors perform in the equilibrium. This equation shows that the equilibrium value of quality depends negatively on s_i. When inventors can capture a larger fraction of the incremental value of their innovations, more inventors will start to perform $R\&D$ until their expected returns are zero. If the policy strength parameter α_i increases, the equilibrium value of quality A_i decreases.

To find the equilibrium aggregate growth rate and the contributions of incumbents and inventors to it, I construct the aggregate innovation supply of incumbents and that of inventors as functions of the value of quality. Here, the aggregate innovation of one group of innovators is the product of their aggregate innovation rate and the step-size of the quality ladder of their innovations.

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Given the structure of the incumbents' value function, the incremental value of incumbent firm j's innovation is given by:

$$V((1+\lambda)q_j) - V(q_j) = \lambda A q_j,$$

which is proportional to A. From equation (23), the solution to the instantaneous optimization problem of a monopolist producer, the innovation rate of an incumbent is given by:

$$x_f(A) = \frac{1}{\delta_f}\lambda A. \tag{27}$$

Because there is a measure one of incumbent firms, the aggregate innovation supply of incumbent firms is

$$x_f(A)\lambda = \frac{1}{\delta_f}\lambda^2 A. \tag{28}$$

The aggregate innovation supplied by incumbents is the amount of innovation on the supply curve corresponding to the equilibrium value of quality:

$$x_f^*\lambda = x_f(A_i)\lambda. \tag{29}$$

The aggregate innovation supplied by incumbents is proportional to the equilibrium value of quality.

To derive the aggregate innovation supply of inventors, consider the optimization problem of a monopoly producer. Given that $I^* = i$, the HJB equation of incumbent firm j is:

$$rAq_j = \pi q_j + x_f(A)\lambda A q_j - c_f(x_f(A))q_j + (1-s_i)\tau_i\mu A q_j. \tag{30}$$

The second term on the right-hand side (RHS) of equation (30) is the expected return to incumbent firm j's $R\&D$.

Both the left-hand side (LHS) and the RHS of the equation are proportional to q_j. Thus, q_j does not affect the relationship of A and $\tau_i\mu$, the aggregate innovation of inventors. $\tau_i\mu$ has two effects on A. The first effect is a firm-level impact through the expected interactions between incumbent firm j and inventors on the market for ideas, or on the related product markets. If $s_i(\alpha_i) \leq 1$, the expected payoffs of inventors are smaller than the incremental value of their innovations, and incumbent firm j can appropriate fraction $(1 - s_i(\alpha_i)) > 0$ of the incremental value of inventors' innovations; as a result, incumbent firm j's equity value increases with τ_i. Specifically, if τ_i increases by $\frac{1}{\mu}$ units, which means $\tau_i\mu$ increases by one unit, the expected benefit of incumbent firm j from inventors' innovations increases by $(1 - s_i(\alpha_i))Aq_j$. This is always the case for market-oriented innovations. If inventors perform entry-oriented $R\&D$, depending on the strength of public policies, $s_e(\alpha_e)$ could be larger or smaller than 1. If $s_e(\alpha_e) > 1$, the aggregate innovation of inventors decreases the equity value

of incumbent firm j. If τ_e increases by $\frac{1}{\mu}$ unit, or $\tau_e\mu$ increases by one unit, the RHS decreases by $(1-s_e(\alpha_e))\tau_e\mu Aq_j$. The firm-level effect of $\tau_i\mu$ on A decreases with s_i.

The second effect is a general equilibrium effect that works through the interest rate r. From Lemma 1, when investors increase their innovation activities, the aggregate growth rate increases. Therefore, the required return on investment, or the interest rate r, also increases. Equation (31) implies that A decreases with r. Intuitively, because r is the factor discounting future profits, keeping everything else constant, a higher interest rate reduces the value of being a monopolist producer. Thus, the general equilibrium effect is always negative.

Substitute r with Equation (10) to rewrite Equation (30) as

$$\tau_i\mu_i s_i(\alpha_i) = \frac{\pi - c_f(x_f(A))}{A} - \rho. \tag{31}$$

The RHS of equation (31) is the ratio of a monopolist's net profit to its equity value, which is usually referred to as the E/P ratio and is negatively related to A. This equation shows the net effect of $\tau_i\mu$ on A, and is negative. It also shows that the scale of the marginal effect increases by $s_i(\alpha_i)$. If $s_i \leq 1$, a one-unit increase in the aggregate innovation of inventors' $\tau_i\mu$ increases the growth rate and the interest rate by one unit. Incumbents internalize fraction $(1 - s_i(\alpha_i))$ of the equity value loss due to the increase in r because they appropriate share $(1-s_i(\alpha_i))$ of the incremental value of inventors' innovations. If $s_i(\alpha_i) > 1$, the expected loss due to the firm-level effect and the general equilibrium effect add together. In the equilibrium, $\tau_i\mu$ pushes down A to the equilibrium level, where the expected net return of inventors is zero.

Rewriting equation (31) yields the aggregate innovation supply of inventors:

$$\tau_i(A)\mu = \frac{1}{s_i(\alpha_i)}(\frac{\pi - c_f(x_f(A))}{A} - \rho). \tag{32}$$

The supply curve is downward sloping, reflecting the negative dependence of A on τ_i. The supply curve shifts downward if s_i increases. Given the E/P ratio, the larger s_i is, the smaller $\tau_i\mu$ is on the supply curve. Intuitively, because the scale of the marginal effect of inventors' aggregate innovation $\tau_i\mu$ on A increases with s_i, the amount of inventors' aggregate innovation needed to push down A to the same level decreases with s_i. The equilibrium aggregate innovation of inventors is $\tau_i(A_i)\mu$, when inventors perform type i $R\&D$.

Proposition 2: In the linear BGP equilibrium, given the equilibrium $R\&D$ choice of inventors $I^* = i$,

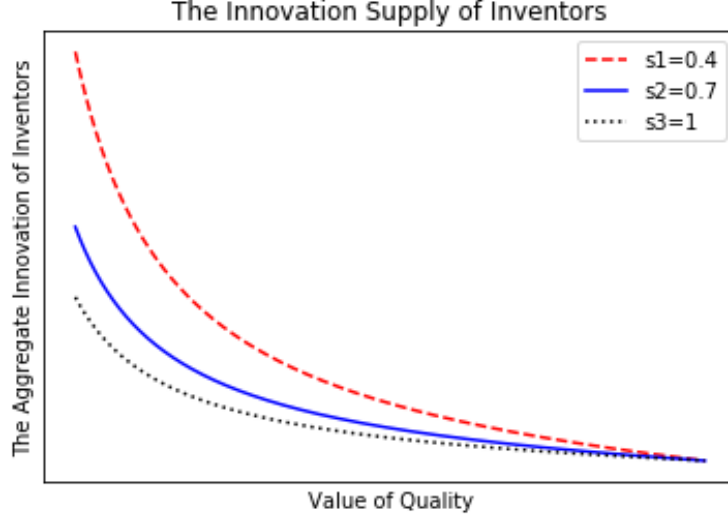$$A_i = \frac{\sqrt{2\phi\delta_i}}{s_i\mu}.$$

Figure 1: The Innovation Supply of Inventors

The aggregate innovation supplied by incumbents is $x_f(A_i)\lambda$, and the aggregate innovation supplied by inventors is $\tau_i(A_i)\mu$.

The proposition shows the complex effects of public policies on innovation. First, policies alter inventors' expected return of different $R\&D$ projects, which affects their $R\&D$ choice and the equilibrium value of quality. Second, policies also change incumbent firms' expected payoffs from interactions with inventors, which changes the slope of the aggregate innovation supply function of inventors.

3   Public Policies and the Aggregate Growth Rate

I separate the value of inventors' bargaining power $s_m(\alpha_m)$ and entrants' appropriability $s_e(\alpha_e)$ into two sets. In one set, inventors' bargaining power in the market for ideas is large enough relative to entrants' appropriability in the product markets, and therefore they choose market-oriented $R\&D$ in equilibrium. In the other set, inventors perform entry-oriented $R\&D$. Public policies can affect innovation incentives of innovators through changing inventors' expected payoffs from their innovations when $s_m(\alpha_m)$ and $s_e(\alpha_e)$ are in a particular set, and also through changing inventors' preferred type of $R\&D$.

A.Public Policies and R&D Investments

I first consider how strengthening public policies affect innovation activities

when inventors' bargaining power and entrants' appropriability are in a particular set. Public polices only change innovation incentives of innovators, but do not change the type of $R\&D$ undertaken by inventors. Assume $I^* = i$, the aggregate growth rate

$$g = x_f(A_i)\lambda + \tau_i(A_i)\mu$$

From Proposition 1, the equilibrium value of the quality level is

$$A_i = \frac{\sqrt{2\phi\delta_i}}{s_i(\alpha_i)\mu}.$$

In equilibrium, inventors' net expected return of type i $R\&D$ is zero, and therefore $s_i A_i \bar{q}$ is the minimum expected payoff at which inventors are willing to innovate. Because a larger $s_i(\alpha_i)$ increases the expected return for any $A > 0$ at $q > 0 A_i$ depends negatively on s_i. Further, with a quadratic $R\&D$ cost function, the relationship is convex.

From equation (27), the aggregate innovation supply of incumbents is a linear function of A_i. Therefore, the equilibrium aggregate innovation supplied by incumbents $x_f(A_i)\lambda$ is also a decreasing and convex function of $s_i(\alpha_i)$.

There are two effects of $s_i(\alpha_i)$ on the aggregate innovation of inventors. The first effect is a positive incentive effect. As $s_i(\alpha_i)$ increases, the expected payoff of inventors also increases. The aggregate innovation supplied by inventors increases to push the value of quality to its new equilibrium level. However, there is also a negative effect via the downward shifting of the innovation supply curve. In equilibrium, $\tau_i(A_i)\mu$ still increases with s_i, but at a decreasing rate.

The aggregate growth rate g reflects the expected technology progress speed from aggregate $R\&D$ of both incumbents and inventors. The marginal influence of $s_i(\alpha_i)$ on g consists of an increasing discouragement effect of $s_i(\alpha_i)$ on incumbents' aggregate innovation and a decreasing encouragement effect on inventors' aggregate innovation, which implies g and $s_i(\alpha_i)$ may have an inverted-U relationship. Define κ_j to be the ratio of the productivity gain of an innovation to the marginal cost parameter:

$$\kappa_f = \frac{\lambda}{\delta_f}, \text{ and } \kappa_i = \frac{\mu}{\delta_j}, j \in \{m, e\}. \tag{33}$$

The following proposition describes the relationship of g and $s_i(\alpha_i)$.

Proposition 3: The aggregate growth rate is a single-peaked function of the proportion $s_i(\alpha_i)$ of the incremental value of its innovation that an inventor can appropriate.

If

$$\frac{\rho\kappa_i}{\kappa_f^2} < 1, i \in \{m, e\} \text{ and } \frac{1}{1 - \frac{\rho\kappa_i}{\kappa_f^2}} < 1 + \frac{1}{\mu}, \tag{34}$$
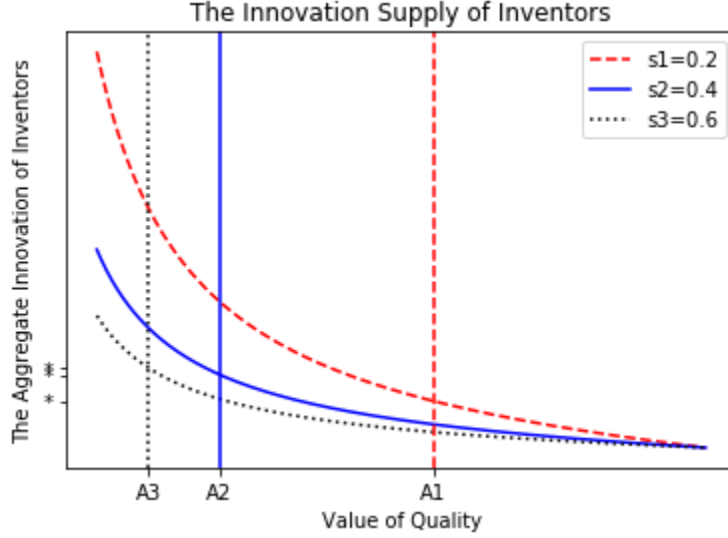
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Figure 2: The Aggregate Innovation of Inventors

in the linear BGP equilibrium, there exists

$$\tilde{s}_i = \frac{1}{1 - \frac{\rho\kappa_i}{\kappa_f^2}} > 1, \tag{35}$$

such that the economic growth rate g increases with $s_i(\alpha_i)$ for $s_i(\alpha_i) \leq \tilde{s}_i$, but decreases with $s_i > \tilde{s}_i$. If

$$\frac{\rho\kappa_m}{\kappa_f^2} \geq 1, \tag{36}$$

then the economic growth rate g increases with $s_i(\alpha_i)$ for all $s_i(\alpha_i) \in [0, 1 + \frac{1}{\mu}]$.

If technological transactions between an inventor and an incumbent only happen ex-post when the innovation is discovered, then $s_m(\alpha_m) \leq 1$ because the incumbent does not pay more than the incremental value of the innovation. Proposition 3 reveals that in this instance, the economic growth rate g always increases with the bargaining power of inventors. If $s_m(\alpha_m)$ increases, the expected payoffs of market-oriented innovations increase, and therefore inventors' investments increase, and more inventors start performing $R\&D$. When inventors' bargaining power increases, incumbent firms' equity value decreases due to expected higher prices for innovations sold on the market for ideas, which reduces their innovation incentives. However, from equation (35), s_m is smaller than the $\tilde{s}_m$. Thus, the encouragement effect of $s_m(\alpha_m)$ on the aggregate innovation of incumbents always dominates the discouragement effect of $s_m(\alpha_m)$ on the aggregate innovation of incumbents. Thus, the economic growth rate depends positively on α_m, the protectiveness of public polices on the market for
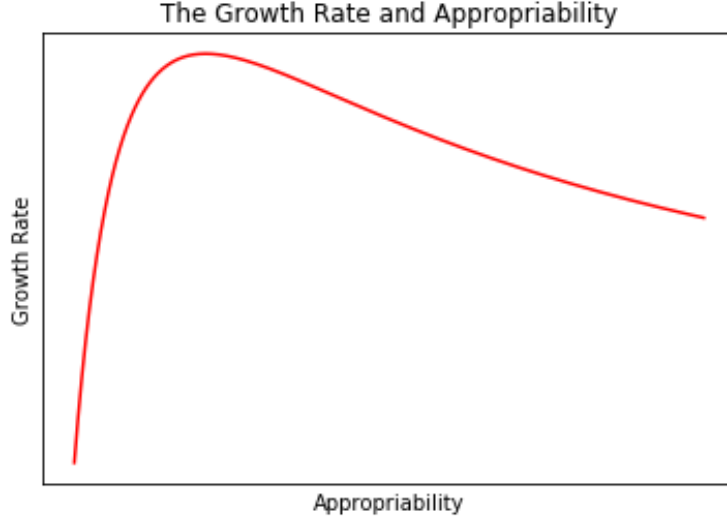
ideas.

Figure 3: The Growth Rate and Entrants' Appropriability

Because $R\&D$ activities of both incumbents and inventors contribute to the growth of the economy, a public policy must balance impacts on their incentives to innovate in order to promote growth. The optimal strength of a public policy that protects entrants from incumbents is determined by the $R\&D$ technologies of incumbents and inventors. If the step-size of the quality improvement of inventors' innovations is sufficiently large, or δ_e is sufficiently small, it is always more important to encourage inventors to innovate than to maintain incumbents' incentives; therefore, public policies should protect entrants. However, when $\tilde{s}_e$ exists, the relationship between the aggregate growth rate of the economy and the strength of a public policy is an inverted-U shape. The optimal strength of a public policy sets entrants' appropriability at $\tilde{s}_e$. As μ increases or δ_e decreases, $\tilde{s}_e$ increases. From equation (23),

$$s_e(\alpha_e) = \frac{\alpha_e(1+\mu)}{\mu},$$

and therefore $s_e(\alpha_e)$ depends negatively on μ. Intuitively, the larger the step-size of the quality improvement of inventors' innovations is, the smaller is the monopoly value of inventors' innovations relative to the incremental value. Therefore the optimal strength should increase with the efficiency of entry-oriented $R\&D$.
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B. Market-Oriented versus Entry-Oriented R&D

I now consider the case in which inventors switch from entry-oriented $R\&D$ to market-oriented $R\&D$. Under Assumption 1, entry-oriented $R\&D$ is more costly relative to market-oriented $R\&D$ for the same innovation rate. Therefore, even when the expected payoffs of entry-oriented innovations exceed that of market-oriented innovations, inventors may prefer market-oriented $R\&D$ projects. Improving inventors' bargaining power thus encourages them to undertake the more cost-efficient type of $R\&D$ projects.

Suppose that the value of inventors' bargaining power $s_m(\alpha_m)$ is fixed at the level where inventors are indifferent to the two types of $R\&D$ projects, which is denoted by s_0. From equation (23),

$$s_0 = s_e(\alpha_e)\sqrt{\frac{\delta_m}{\delta_e}}. \tag{37}$$

Because entry-oriented $R\&D$ is more costly than market-oriented $R\&D$, i.e., $\delta_e > \delta_m$, $s_0 < s_e(\alpha_e)$, when inventors are indifferent to the two types of $R\&D$ projects, they capture a large share of the incremental value of their innovations through entry-oriented $R\&D$ compared with market-oriented $R\&D$. Comparing the supply curve of the two types of innovation of inventors, the supply curve of market-oriented innovation lies above the supply curve of entry-oriented innovation. This is because the net effect of $\tau_e\mu$ on the E/P ratio of a monopolist is smaller than that of $\tau_m\mu$. In order to push the E/P ratio to the same level, $\tau_m\mu$ must exceed $\tau_e\mu$ when the E/P ratio is positive.

Compare two equilibria when $s_m = s_0$. In one equilibrium, inventors perform market-oriented $R\&D$ selling their innovations to incumbent firms; while in the other equilibrium, they perform entry-oriented $R\&D$. With equation (24), the value of quality in the two equilibra is the same:

$$A_m = \frac{\sqrt{2\phi\delta_m}}{s_0\mu} = A_e = \frac{\sqrt{2\phi\delta_e}}{s_e\mu}.$$

In the two equilibria, the aggregate innovation of incumbents is the same,

$$x_f(A_m)\lambda = x_f(A_e)\lambda.$$

However, the aggregate innovation of inventors differs. Because $s_0 < s_e(\alpha_e)$, incumbent firms capture a smaller fraction of the incremental value of inventors' innovations through competition with entrants in product markets than through purchasing innovations from inventors. When aggregate type m innovation rate τ_m and aggregate type e innovation rate τ_e are the same, incumbent firms' equity value is lower due to smaller expected payoffs from interactions with inventors. Thus, on the two aggregate innovation supply curves of inventors, at the same value of quality, the aggregate innovation supplied by inventors
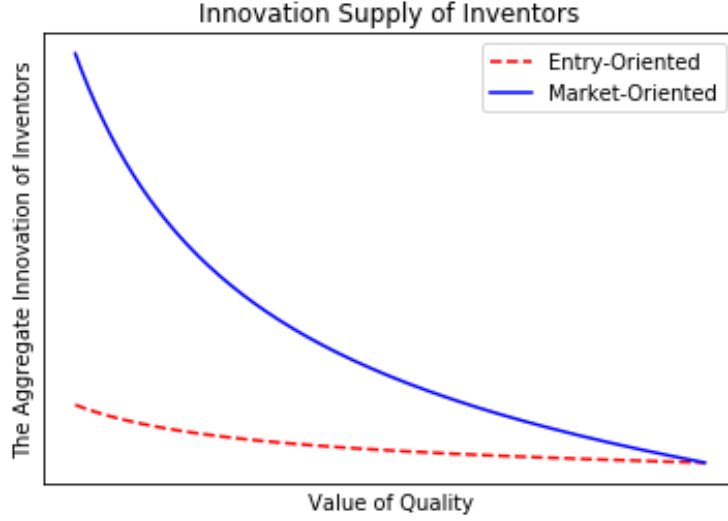
Innovation Supply of Inventors

Figure 4: The Supply of Market-Oriented Innovation vs. Entry-Oriented Innovation

in the market-oriented $R\&D$ equilibrium exceeds the aggregate innovation supplied by inventors in equilibrium when inventors undertake entry-oriented $R\&D$.

Proposition 4: If inventors' bargaining power is s_0, the growth rate of the equilibrium where inventors perform type M $R\&D$ is higher than when inventors perform entry-oriented $R\&D$.

Proposition 4 highlights the important role that patents play as the market foundation of technology transactions. Stronger IPR protection for inventors from incumbent firms increases their bargaining power on the market for ideas. If inventors' bargaining power is weak, they choose to perform entry-oriented $R\&D$ projects. If, instead, IPR is strong, inventors undertake market-oriented $R\&D$ projects. Entry-oriented $R\&D$ more heavily depresses the value of being an incumbent than does market-oriented $R\&D$. On the aggregate innovation supply curves of inventors, at the same value of quality, aggregate type m innovation exceeds aggregate type e innovation. When inventors' bargaining power is at the level that they are indifferent between the two types of $R\&D$, the equilibrium value of quality does not change with the choice of $R\&D$, which implies that aggregate innovation supplied by incumbent firms is the same, but that aggregate innovation supplied by inventors is higher when inventors perform market-oriented $R\&D$ than when they perform entry-oriented $R\&D$. When inventors switch to market-oriented $R\&D$ projects, the result is faster economic growth. Propositions 3 and 4 together indicate that strong IPR protection for inventors' innovations always spurs the economy.
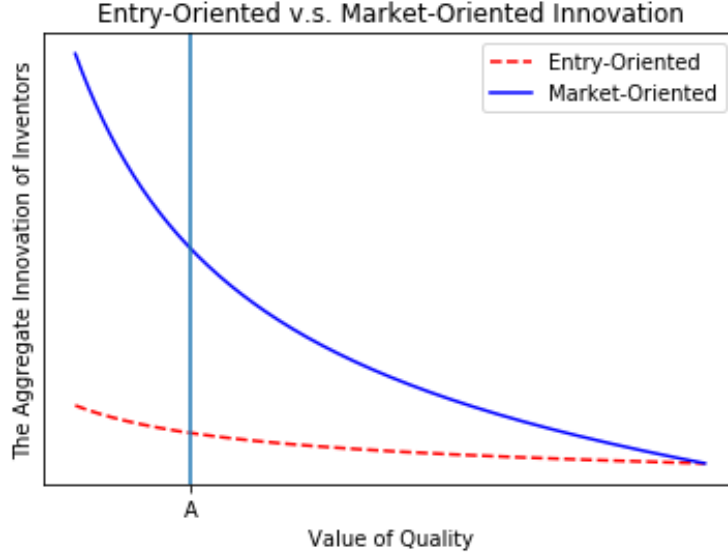
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Figure 5: Market-Oriented vs. Entry-Oriented $R\&D$

Proposition 4 also implies that if entrants' appropriability decreases to a level such that $s_e(\alpha_e) = s_m(\alpha_m)\sqrt{\frac{\delta_e}{\delta_m}}$, then the aggregate growth rate increases.

4 Public Policies and the Allocation of Monopoly Value

Now I discuss the relationships between the protectiveness of public policies and how incumbents and inventors share monopoly value when they interact in the market for ideas or in the product markets. First, I show why strengthening IPR protection against imitation by incumbent firms increases inventors' bargaining power on the market for ideas. Then, I discuss policies protecting entrants on product markets.

Patent Strength and Inventors' Bargaining Power. A critical factor that determines inventors' bargaining power on the market for ideas is the protection strength of their IPR. As discussed in Gans and Stern (2003), when selling innovations, inventors often face the "paradox of disclosure" problem. Incumbent firms' willingness to pay depends on their knowledge of how innovations on the market for ideas can improve their profits. However, information disclosure by inventors increases the ability of incumbent firms to imitate these innovations, thereby reducing their willingness to pay for these innovations. Stronger IPR protection increases inventors' bargaining power by reducing the ability of

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incumbent firms to invent around inventors' patents. I develop a bargaining process with the "paradox of disclosure" problem to capture the importance of IPR protection on the 'market for ideas.

Consider a transaction between one inventor and one monopoly producer. The inventor has to reveal some particular features of the innovation to the buyer because of asymmetric information on the innovation's quality[3]. The buyer cannot back-engineer the invention initially. After accumulating more knowledge through inspection, with probability $1 - \alpha_m \in (0, 1)$, the buyer can imitate without infringement; and with probability α_m, the buyer cannot imitate. If the buyer cannot imitate, it pays the inventor at a price given by the Nash bargaining solution. The parameter α_m is positively related to the strength of the inventor's patent.

In equilibrium, the inventor receives

$$\frac{\alpha_m}{2}(V((1 + \mu)q_j(t)) - V(q_j(t)))$$
(38)

The inventors' bargaining power reflects the patent strength α of the inventor's innovation according to

$$s_m = \frac{\alpha_m}{2} \in [0, \frac{1}{2}]$$
(39)

Public Policies and Appropriability of Type E Innovations. In this paper, I consider antitrust policies that restrict predatory activities and IPR policies that limit the blocking power of incumbent firms' patents. The model described in Section 2 is used to identify impacts of antitrust policies that increase the survival probability of entrants by restricting the predatory activity of incumbent firms. Now I introduce another important policy and show how the model could easily be modified to apply to it.

Policy 2: An IP policy to constrain blocking patents. In innovative industries, innovation is often cumulative. New products introduced by entrants are sometimes enhancements of incumbents' products. If an incumbent holds a broad patent, an entrant to the product market may be not able to commercialize its innovation without a license from the incumbent because the incumbent can sue the entrant for patent infringement. At the same time, the incumbent cannot produce the new product because the improved feature is covered by the entrant's patent. In that case, the entrant and the incumbent must negotiate a license contract in order to use the entrant's innovation. The scope of incumbents' patents determines the probability with which a court considers improved

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[3]Assume that there are a large number of imposters who hold low-quality innovations that do not improve the quality of the buyer' products. The buyer cannot differentiate the inventor from among the imposters based on the limited information revealed by patents. Therefore, that inventor's optimal strategy is to allow the buyer to inspect and resolve information asymmetries.

products to infringe upon those patents. A stronger IP policy can increase the payoff inventors expect by restricting the scope of patents held by incumbents.

To model this, I assume that the monopoly producer of every intermediate good holds a patent on the current product. An incumbent and an entrant can negotiate a license contract to use the incumbent's patent in the shadow of litigation. If the entrant does not purchase a license from the incumbent, with probability $1 - \alpha_e \in [0, 1]$, the incumbent can successfully sue the entrant for patent infringement. Under that condition, incumbents continue producing goods at their original level of quality, and the entrant exits the product market. Because the relevant intermediate good is not produced at the highest quality, there is an efficiency loss. With the expectation of the efficiency loss from a subsequent lawsuit, the two parties prefer to bargain before lawsuits over a license fee for the incumbent's patent. To avoid complex strategic incentives, I focus on bargaining outcomes with lump-sum payments.

Consider a bargaining process between incumbent firm j and an entrant to settle the lawsuit of patent infringement. With probability $1 - \alpha_e$, the incumbent wins the lawsuit and maintains its monopoly profit. Its expected payoff is

$$(1 - \alpha_e)V(q_j).$$

The entrant's expected payoff if bargaining breaks up is

$$\alpha_e V((1 + \mu)q_j)$$

Using the Nash bargaining solution, the license fee to monopolist j is

$$\frac{1 - \alpha_e}{2}(V((1 + \mu)q_j) - V(q_j(t))) + (1 - \alpha_e)V(q_j), \tag{40}$$

and the payoff of to the entrant is

$$R_e(\alpha_e, q_j) = \frac{1 - \alpha_e}{2}(V((1 + \mu)q_j) - V(q_j)) + \alpha_e V((1 + \mu)q_j). \tag{41}$$

There is no asymmetric information about the quality of the entrant's technology in the bargaining process because it has successfully brought the new product to the product market. Given the equity value function of a monopolist $V(\cdot)$, $R_e(q_j)$ increases with α_e for any intermediate good $j \in [0, 1]$. When $\alpha_e = 1$, the entrant captures all of the monopoly value of its innovation.

Because $R_e(\alpha_e, q_j)$ is a weighted sum of the monopoly value and incremental value of the innovation, when the value function takes the form $V(q) = Aq$ for any $q > 0$, the fraction of incremental value that the entrant appropriates is

$$s_e(\alpha_e) = \frac{1 - \alpha_e}{2} + \alpha_e \frac{1 + \mu}{\mu}. \tag{42}$$

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$s_e(\alpha_e)$ depends positively on the strength of the policy α_e. It is a decreasing function of μ. Intuitively, the larger the incremental value of inventors' innovations, the smaller the monopoly value of those innovations to their incremental value.

Because the model in Section 2 only requires $s_e(\alpha_e)$ to be exogenous in the linear BGP equilibrium, it can be used directly to analyze Policy 2. According to Proposition 3, the aggregate growth rate g is a single peaked function of $s_e(\alpha_e)$. If entry-oriented $R\&D$ is efficient enough relative to incumbents' $R\&D$, g increases with $s_e(\alpha_e)$; this implies that stronger restrictions on incumbent firms' blocking patents always fosters faster economic growth. When $\tilde{s}_e$ exists, there exists an optimal protectiveness of entrants that sets $s_e(\alpha_e)$ to $\tilde{s}_e$. Because $s_e(\alpha_e)$ is a decreasing function of μ, and $\tilde{s}_e$ is an increasing function of μ, the optimal strength of the policy depends positively on μ.

5 Directed Entry

In the baseline model, inventors only know the application of their innovations when they discover them. The commercialization methods of those innovations are given by ex-ante $R\&D$ choice. In reality, inventors often plan to innovate to enter product markets or sell to incumbents after they have a research idea. I now modify the baseline model to allow inventors to know the applications of their innovations before they choose $R\&D$ projects. I first develop the model and show that results from the baseline model still hold. I then allow incumbent firms to contract with inventors to perform $R\&D$. The information structure here provides insights into contract $R\&D$ and $R\&D$ cooperation between incumbents and inventors, which are common in practice.

I now assume that if an inventor decides to perform $R\&D$, it enters a setup stage and randomly draws an idea that is applicable to one intermediate good $j \in [0,1]$ with a uniform distribution. The inventor can develop the idea into a market-oriented (type m) innovation and sell it to the incumbent or into an entry-oriented (type e) innovation and enter the product market. The inventor can also forfeit the idea and exit.

If an inventor invests in type i $R\&D$, it receives an innovation with a flow rate $\frac{\psi}{\sqrt{\delta_i}}$ by spending wq_j units of the final good on $R\&D$, where $i \in \{m,e\}$, and $\psi > 0$, $w > 0$ and δ_i are constant parameters. Stating that the $R\&D$ cost is proportional to q_j implies that it is more expensive to improve higher-quality products. I assume $\delta_e > \delta_m$ because entry-oriented innovations require larger costs to develop compared with market-oriented innovations. I assume ψ is small enough that an inventor takes the aggregate type i (i{m,e}) innovation rate for every intermediate good as given. In other words, an inventor does not consider the effect of its innovation on the equity value of an incumbent firm, or $R\&D$ decisions of all other innovators in the economy.

Inventors' Optimization Problem If an inventor invests in $R\&D$, it chooses the type of $R\&D$ that is expected to yield the largest return. Consider an inventor whose idea is applicable to intermediate good j. By investing w units of the final goods in market-oriented $R\&D$, the expected net return to the inventor is given by:

$$v_j(m) \equiv \frac{\psi}{\sqrt{\delta_m}} s_m(V((1+\mu)q_j) - V(q_j)) - wq_j.$$

If the inventor invest w units of the final good in entry-oriented $R\&D$, its expected net return is given by:

$$v_j(e) \equiv \frac{w}{\sqrt{\delta_e}} R_e(\alpha_e, q_j) - wq_j.$$

The inventor invest in $R\&D$ if and only if its optimal $R\&D$ return is at least wq_j:

$$\max\{v_j(m), v_j(e))\} \geq wq_j$$

If investing in $R\&D$, the inventor chooses market-oriented $R\&D$ if and only if

$$v_j(m) \geq v_j(e) \tag{43}$$

Inventors' expected net return before they draw ideas from the uniform distribution is given by

$$E_{j\in[0,1]} \max\{v_j(m), v_j(e), 0\}$$

Then, inventors' free-entry condition is given by:

$$E_{j\in[0,1]} \max\{v_j(m), v_j(e), 0\} = 0$$

Let M_j denote the measure of inventors who perform $R\&D$ for intermediate good j. If all M_j inventors perform type i innovation, then the aggregate type i innovation rate for intermediate good j is

$$\tau_{i,j} = M_j \frac{\psi}{\sqrt{\delta_i}}. \tag{44}$$

For incumbent firm j whose technology level is q_j, the object is to choose the Poisson arrival rate of its innovation conditional on $\tau_{m,j}$, $\tau_{e,j}$ and the interest rate r. Its Hamilton-Jacobi-Bellman equation is the same as that in the baseline model:

$$rV(q_j) - \dot{V}(q_j) = \pi q_j + \max_{x_f}\{x_f[V((1+\lambda)q_j) - V(q_j)] - c_f(x_f)q_j\}$$

$$+\tau_{m,j}(1 - s_m(\alpha_m))[V((1+\mu)q_j) - V(q_j)] + \tau_{e,j}[V((1+\mu)q_j) - R_e(\alpha_e, q_j) - V(q_j)] \tag{45}$$

Now I define the BGP equilibrium of the extension. In addition to the definition in the baseline model, I add the requirements for the empirical distribution of inventors' ideas.

Definition (BGP equilibrium): A BGP equilibrium is a BGP equilibrium of the baseline model that satisfies the following condition: the measure of inventors performing $R\&D$ for intermediate good $j \in [0, 1]$ is constant over time.

I only consider the linear BGP equilibrium, where the value function of a monopolist takes the form $V(q) = Aq$. Using the linear form of the value function, the expected net return of type i ($i \in \{m, e\}$) $R\&D$ for intermediate good j is given by:

$$v_j(i) = \frac{\psi}{\sqrt{\delta_i}} s_i(\alpha_i) \mu A q_j - w q_j, \tag{46}$$

where $s_i(\alpha_i)$ is the fraction of the incremental value that inventors receive from their innovations. This equation implies that the sign of $v_j(i)$ is the same for all intermediate goods. Thus, if inventors are willing to invest in $R\&D$ for any intermediate good, they will invest in all of the intermediate goods. $v_j(m) \geq v_j(e)$ if and only if $\frac{s_m}{s_e} \geq \sqrt{\frac{\delta_m}{\delta_e}}$, which mirrors the baseline setting. Therefore, inventors choose market-oriented $R\&D$ if and only if this holds.

With no aggregate uncertainty, for a time interval of length dt, the measure of inventors who have started performing $R\&D$ during dt for a particular intermediate good is the same for all intermediate goods. Assume the equilibrium $R\&D$ choice of inventors $I^* = i$. Then the measure of inventors who have successfully invented innovations applicable to intermediate good j is $M_j \frac{\psi}{\sqrt{\delta_i}} dt$. M_j remains the same over time if and only if the measure of inventors who started performing $R\&D$ during dt equals the measure of inventors who exit. Thus, the linear BGP, M_j is the same for all intermediate goods. From equation (44), the aggregate type i innovation rate for intermediate good j is the same for all intermediate goods. From now on, I drop the subscript j in M_j and $\tau_{i,j}$. Adding up the measure of inventors, and aggregate type i innovation rate for all intermediate goods, the measure of inventors performing $R\&D$ equals M, and the aggregate type i innovation rate equals τ_i.

With the same process as in the baseline model, I derive the solution to the linear BGP equilibrium of the extension.

Proposition 5. The linear BGP equilibrium is unique and is characterized by: the optimal $R\&D$ strategy $I^* = i \in \{m, e\}$ of inventors; the equilibrium value of quality; and the aggregate innovation supply functions of incumbents and inventors, which satisfies the following conditions:
(1) $I^* = m$ if and only if

$$\frac{s_m(\alpha_m)}{s_e(\alpha_e)} \geq \sqrt{\frac{\delta_m}{\delta_e}}.$$

Given $I = i$,

(2) $A_i = \frac{w\sqrt{\delta_i}}{\psi s_i \mu}$;

(3) The aggregate innovation supply functions of incumbents and inventors are the same in the baseline model;

(4) The measure of inventors who actively perform R&D is

$$M_i = \frac{\sqrt{\delta_i}\tau_i}{\psi}. \tag{47}$$

Proposition 5 implies that the equilibrium of this model has the same properties as the equilibrium of the baseline model if there is no ex-ante contracting. I now show that the possibility of ex-ante R&D contracts between incumbents and inventors can increase the growth rate.

Assume that an incumbent firm can contract with inventors whose ideas are applicable to its good at every moment of time. At time t, incumbent firm j makes a take-it-or-leave offer to inventors that specifies a payment $R_m(q_j)$ for a market-oriented innovation at time $t + dt$. Because the application of an inventor's idea is randomly drawn from a uniform distribution, this offer does not change either the entry decisions by inventors or the measure of inventors performing R&D for intermediate good j. If inventors already prefer market-oriented R&D, monopolist j cannot increase their investment by proposing an offer $P(q_j)$ more than their expected payoffs from ex-post transactions. Thus monopolist j only contracts with them to deter entry-oriented R&D.

Inventors accept the offer if its expected net payoff from the contract is larger than the expected net payoff from entry-oriented R&D, which implies $P(q_j)$ must satisfy:

$$\frac{\psi}{\sqrt{\delta_m}}P(q_j) \geq \frac{\psi}{\sqrt{\delta_e}}R_e(q_j). \tag{48}$$

Incumbent firm j is willing to propose the contract if its expected net payoff is better than that of letting inventors perform type E R&D, and therefore $P(q_j)$ satisfies:

$$\frac{\psi}{\sqrt{\delta_m}}V((1+\mu)q_j) - P(q_j) - V(q_j)) \geq \frac{\psi}{\sqrt{\delta_e}}(V((1+\mu)q_j) - R_e(q_j) - V(q_j)). \tag{49}$$

Here $V(q_j)$ is the value of monopolist j conditional on its optimal strategies including the option of ex-ante contracting. From equation (47), if $P(q_j)$ exists, it is given by:

$$P(q_j) = \sqrt{\frac{\delta_m}{\delta_e}}R_e(q_j). \tag{50}$$

Incumbent firm j is better off by offering inventors the contract if and only if equation (48) is satisfied. Plugging equation (49) in equation (48) yields:

$$\frac{\psi}{\sqrt{\delta_m}}V((1+\mu)q_j) - V(q_j)) \geq \frac{\psi}{\sqrt{\delta_e}}(V((1+\mu)q_j) - V(q_j)), \tag{51}$$

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which always holds because $\delta_e > \delta_m$. In other words, when incumbents and inventors can contract $R\&D$ ex-ante, because entry-oriented $R\&D$ is less efficient, incumbents can always contract with inventors to perform market-oriented $R\&D$.

In the linear BGP equilibrium of the model with ex-ante contracting,

$$P(q_j) = s_e \sqrt{\frac{\delta_m}{\delta_e}} \mu A. \tag{52}$$

With this contract, the fraction of market-oriented innovations' incremental value capture by inventors is $s_e \sqrt{\frac{\delta_m}{\delta_e}} < s_e$.

Analysis similar to Proposition 4 implies that the growth rate of the economy increases if incumbents and inventors can sign ex-ante $R\&D$ contracts. Moreover, incumbents and entrants always prefer $R\&D$ contracts as long as market-oriented $R\&D$ is less costly than entry-oriented $R\&D$.

6 Conclusion

In this article, I analyze the interaction between the innovation activities of incumbents and those of inventors. Further, I examine how public policies that affect these activities can be used to spur economic growth. A public policy can shift the distribution of incentives to innovate between incumbents and inventors by altering the allocation of the monopoly value of innovations. To promote fast economic growth, my analysis has found, an optimal share of the monopoly value of inventors' innovations should be allocated to them; the size of that share depends on the research technologies of different innovators.

I highlight the importance of the market for ideas in fostering aggregate innovation. The market for ideas encourages inventors to innovate toward a mutually beneficial objective and fosters technological transactions relative to the strategy of innovating for entry. For inventors, the $R\&D$ is more cost-efficient; for incumbents, their expected payoff from a transaction on the market for ideas is larger than their expected payoff when facing an entrant in their product markets. If IPR protection increases and inventors switch to innovate for the market for ideas, the IPR policy increases inventors' innovation incentive but does not decrease incumbents' innovation incentive, which leads to a faster rate of economic growth.

Due to the tractability problem, I do not intend to explain the role of market structure in the current framework. Incumbents and entrants in the model engage in "competition for the market" rather than "competition in the market." I capture an important competition type, given that in many innovative industries a leader is dominating the product market, and entrants usually compete with the leader for market leadership. I leave the effect of market structure on

economic growth to future research.

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Appendices

Proof of Proposition 3:

Take the derivative of g with respect to s_i:

$$\frac{dg}{ds_i} = \lambda\frac{dx_f(A_i)}{ds_i} + \mu\frac{d\tau_i}{ds_i}$$

$$= \lambda\frac{dx_f(A_i)}{ds_i} - \frac{c_f'(dx_f(A_i))}{s_iA_i}\frac{dx_f(A_i)}{ds_i} + \frac{\rho}{s_i^2}$$

$$= \lambda(1-\frac{1}{s_i})\frac{dx_f(A_i)}{ds_i} + \frac{\rho}{s_i^2}$$

In the second line of equation (31), s_iA_i does not depend on s_i according to equation (23). In the third line of the equation, I use the first order condition of incumbents' optimization problem that $c_f'(dx_f(A_i)) = \lambda A_i$. Therefore as long as $s_i \leq 1$, the positive effect of an increase in s_i on $\tau_i\mu_i$ is greater than the reduction on $dx_f(A_i)\lambda$, and the the rate of economic growth increases with s_i. The result does not depend on the quadratic form of $c_f(\cdot)$.

Plug in the quadratic form of the cost function, I get proposition 3.